On p-conjugacy separability of fibre products

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- $\mathcal{C} = \{$all finite p-gps$\}$, for some prime p. Then we get residually p-finite (Res-p) and p-conjugacy separable (p-CS) gps.

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– the mod-p Torelli gp. of G, i.e., $Out_p(G)$ is the kernel of the action of $Out(G)$ on the mod-p abelianization of G.

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Remark

If G is f.g. then $|Out(G) : Out_p(G)| < \infty$.

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Additionally, the class of p-CS gps. is closed under

- retracts
- direct products
- free products
- graph products (Ferov)

p-CS or not p-CS?

F.g. torsion-free nilpotent gps. are Res-p for all p (Gruenberg).
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Theorem (Ivanova)

If a torsion-free nilpotent gp. G is p-CS (for some prime p) then G is abelian.

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Are limit gps (f.g. fully res. free gps.) p-CS for all p?

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Theorem (Wise)

If G is a limit group then there exist $H \leqslant_f G$, a RAAG A and $K \leqslant_f A$ s.t. H is a retract of K.

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To answer the above questions one can start with the simplest interesting case, when the RAAG is the direct product of two free groups.

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If $\ker \psi \neq \{1\}$ then G is a full subdirect sbgp. of $F_1 \times F_2$ and $F_1/(G \cap F_1) \cong P$.

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Theorem (M.)

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- G_6 is not p-CS for any prime p (as $\mathbb{Z}/6\mathbb{Z}$ is not a p-gp.).

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If A is a RAAG whose defining graph contains an induced square then A has a sbgp. of index 6 which is not p-CS for any prime p.

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If A is a RAAG whose defining graph contains an induced square then A has a sbgp. of index 6 which is not p-CS for any prime p.

Question

Let Γ be the path of length 3. Does the RAAG $A = A(\Gamma)$ have non-(p-CS) f.i. sbgps.?

Theorem (M.)

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Corollary

Suppose that F_1, F_2 are free gps. and $G \leqslant F_1 \times F_2$ is any sbgp. If G is p-CS for at least two distinct primes p, then G is itself a direct product of free gps.

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- *$\exists$ infinite finitely presented RF p-group;*
- *$\exists$ a f.g. sbgp. $G \leqslant \mathbb{F}_2 \times \mathbb{F}_2$ s.t. G is p-CS and G is not virtually a direct product of two free gps.*

Some corollaries

Theorem (M.)

Let $G \leqslant F_1 \times F_2$ be a full subdir. sbgp, where F_i is a non-abelian free gp., $i = 1, 2$, and let $P = F_1/(F_1 \cap G)$. TFAE:

- *G is p-CS;*
- *P is a p-group and P is res. finite.*

Corollary

Let p be any prime. TFAE:

- *$\exists$ infinite finitely presented RF p-group;*
- *$\exists$ a f.g. sbgp. $G \leqslant \mathbb{F}_2 \times \mathbb{F}_2$ s.t. G is p-CS and G is not virtually a direct product of two free gps.*

It is presently unknown whether there exist infinite f.p. RF *p*-gps. (Grigorchuk's group is an example of an infinite f.g. RF 2-gp.).